

Enhanced

# Worksheet 5.1A

## Area of a Triangle $\left(\frac{1}{2}ab \sin C\right)$

NF

(Refer to Book 5A Ch5 p. 5.6 – 5.13)

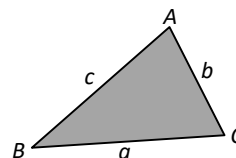
Name: \_\_\_\_\_

Class: \_\_\_\_\_

### Key Points

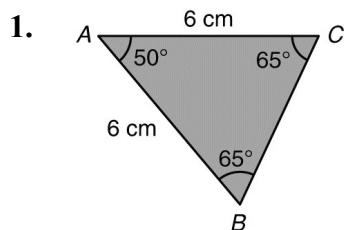
Given two sides and their included angle

$$\text{Area of } \triangle ABC = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$$

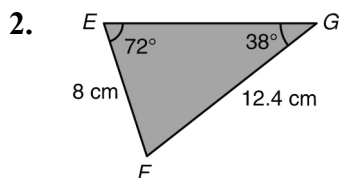


(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

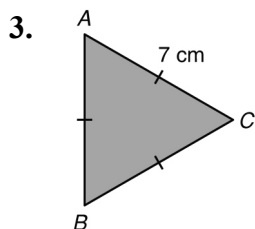
Find the areas of the following triangles. (1 – 3)



Solution



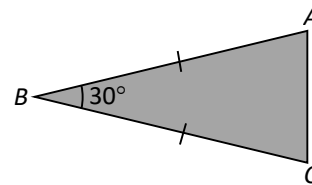
Solution



Solution

4. In the figure,  $AB = BC$  and  $\angle ABC = 30^\circ$ . If the area of  $\triangle ABC$  is  $24 \text{ cm}^2$ , find the length of  $AB$ .

**Solution**

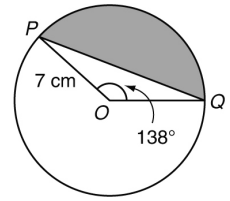


5. The area of  $\triangle PQR$  is  $60 \text{ cm}^2$ . If  $PQ = 12 \text{ cm}$  and  $QR = 14 \text{ cm}$ , find all possible values of  $\angle PQR$ .

**Solution**

6. In the figure,  $O$  is the centre of the circle.  $OP = 7$  cm and  $\angle POQ = 138^\circ$ .

Find the area of the shaded region.



**Solution**

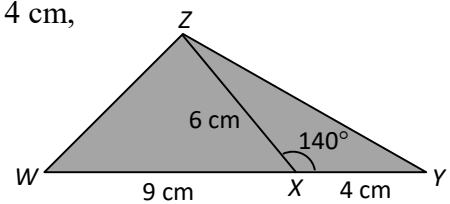
Area of the shaded region

= area of sector  $POQ$  – area of  $\triangle POQ$

$$= \left[ \frac{(\quad)}{360^\circ} \times \pi \times (\quad)^2 - (\quad) \right] \text{cm}^2$$

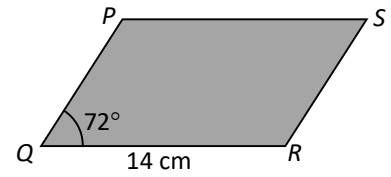
=

7. In the figure,  $X$  is a point on  $WY$  such that  $WX = 9$  cm,  $XY = 4$  cm,  $XZ = 6$  cm and  $\angle YXZ = 140^\circ$ . Find the area of  $\triangle WYZ$ .



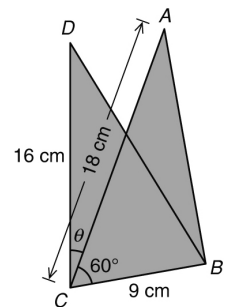
**Solution**

8. In the figure,  $PQRS$  is a parallelogram with  $QR = 14$  cm and  $\angle PQR = 72^\circ$ . If the area of the parallelogram is  $40 \text{ cm}^2$ , find the length of  $PQ$ .



**Solution**

9. In the figure,  $BC = 9$  cm,  $AC = 18$  cm,  $CD = 16$  cm and  $\angle ACB = 60^\circ$ . If the areas of  $\triangle ABC$  and  $\triangle DBC$  are equal, find all possible values of  $\theta$ .



**Solution**

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# Worksheet 5.1B

## Area of a Triangle (Heron's Formula) NF

(Refer to Book 5A Ch5 p. 5.14 – 5.18)

Name: \_\_\_\_\_

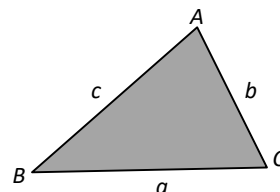
Class: \_\_\_\_\_

### Key Points

Given three sides

Area of  $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$ , where  $s = \frac{a+b+c}{2}$ .

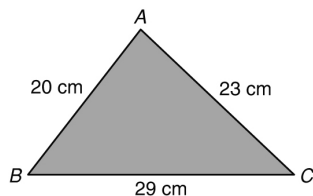
This formula is known as Heron's formula.



(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

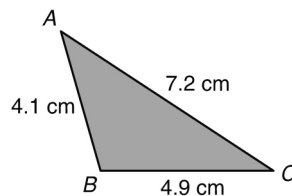
Find the areas of the following triangles. (1 – 2)

1.



**Solution**

2.



**Solution**

3. In the figure,  $AB = 15$  cm,  $BC = 25$  cm and  $CD = 30$  cm.

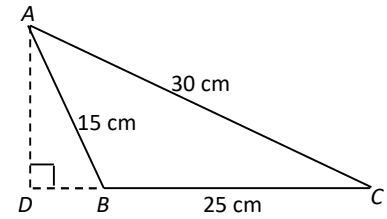
(a) Find the area of  $\triangle ABC$ .

(b) Find the length of  $AD$ .

**Solution**

(a)

(b)



4. The perimeter of  $\triangle ABC$  in the figure is 60 m.

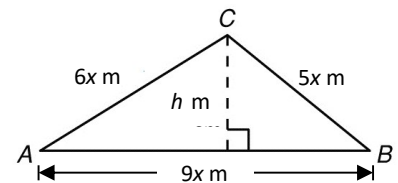
(a) Find the value of  $x$ .

(b) Find the value of  $h$ .

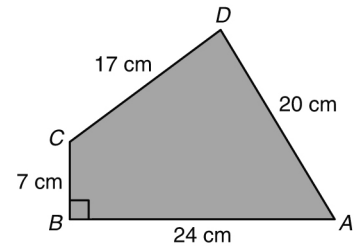
**Solution**

(a)

(b)



5. In the figure,  $AB = 24$  cm,  $BC = 7$  cm,  $CD = 17$  cm,  $AD = 20$  cm and  $BC \perp AB$ .



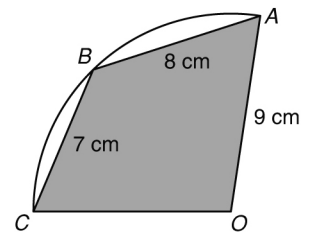
- (a) Find the length of  $AC$ .  
(b) Find the area of quadrilateral  $ABCD$ .

**Solution**

(a)

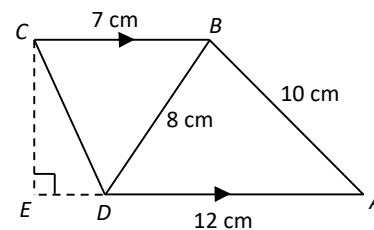
(b)

6. In the figure,  $OAC$  is a sector of radius 9 cm and centre  $O$ . If  $AB = 8$  cm and  $BC = 7$  cm, find the area of quadrilateral  $OABC$ .



**Solution**

7. In the figure,  $ABCD$  is a trapezium with  $AD \parallel BC$ ,  $AB = 10$  cm,  $BC = 7$  cm,  $BD = 8$  cm and  $AD = 12$  cm.



- (a) Find the area of  $\triangle ABD$ .
- (b) (i) Find the length of  $CE$ .
- (ii) Find the area of  $\triangle BCD$ .

**Solution**

(a)

(b) (i)

(ii)



Basic

# Worksheet 5.2 The Sine Formula

NF

(Refer to Book 5A Ch5 p. 5.19 – 5.28)

Name: \_\_\_\_\_

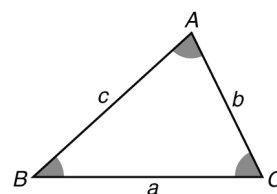
Class: \_\_\_\_\_

## Key Points

### The sine formula

In  $\triangle ABC$ ,

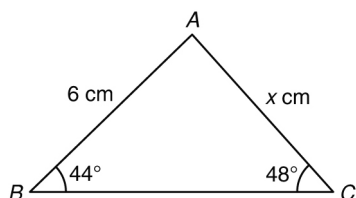
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

Find the values of  $x$  in the following triangles. (1 – 4)

1.



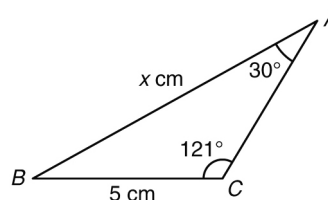
**Solution**

By the sine formula,

$$\frac{AC}{\sin B} = \frac{(\quad)}{\sin (\quad)}$$

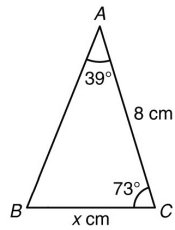
$$\frac{x \text{ cm}}{\sin 44^\circ} =$$

2.



**Solution**

3.

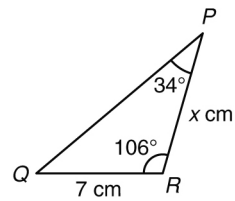


**Solution**

$$A + B + C = 180^\circ \quad (\angle \text{ sum of } \triangle)$$

$$(\quad)^\circ + B + (\quad)^\circ = 180^\circ$$

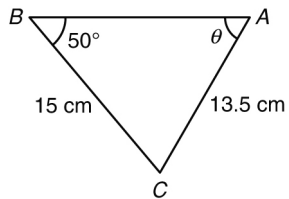
4.



**Solution**

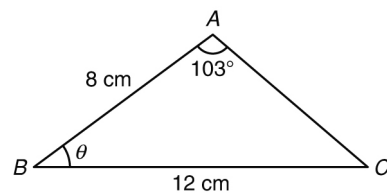
In each of the following triangles,  $\theta$  is an acute angle. Find  $\theta$ . (5 – 6)

5.



**Solution**

6.



**Solution**

Find  $B$  in  $\triangle ABC$  under each of the following conditions. (7 – 8)

7.  $A = 55^\circ$ ,  $a = 13$  cm,  $b = 15$  cm

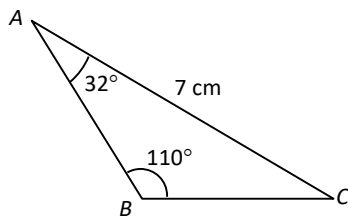
**Solution**

8.  $A = 155^\circ$ ,  $a = 9$  m,  $b = 16$  m

**Solution**

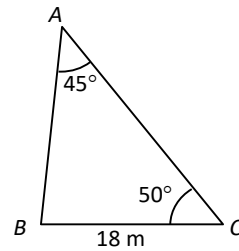
Solve  $\triangle ABC$  under the following conditions. (9 – 10)

9.



**Solution**

10.



**Solution**

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# Worksheet 5.2 The Sine Formula

NF

(Refer to Book 5A Ch5 p. 5.19 – 5.28)

Name: \_\_\_\_\_

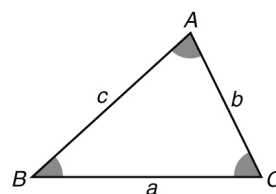
Class: \_\_\_\_\_

## Key Points

### The sine formula

In  $\triangle ABC$ ,

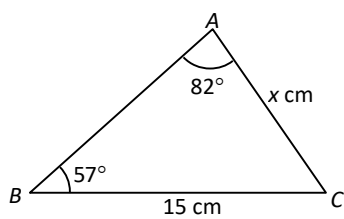
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

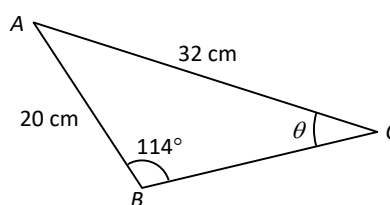
Find the unknowns in the following triangles. (1 – 2)

1.



**Solution**

2.



**Solution**

Find the unknown side/angle of  $\triangle ABC$  in brackets under each of the following conditions. (3 – 4)

3.  $B = 70^\circ$ ,  $C = 45^\circ$ ,  $c = 11$  cm [a]

**Solution**

4.  $B = 120^\circ$ ,  $a = 3\sqrt{3}$  m,  $b = 9$  m [C]

**Solution**

Solve  $\triangle ABC$  under the following conditions. (5 – 6)

5.  $B = 65^\circ$ ,  $b = 20$  cm,  $c = 14$  cm

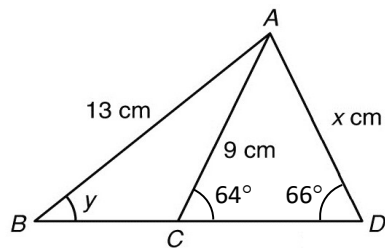
**Solution**

6.  $A = 36^\circ$ ,  $a = 12$  cm,  $c = 18$  cm

**Solution**

Find the unknowns in the following figures. (7 – 8)

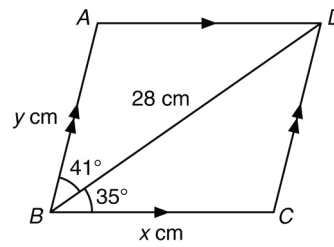
7.



$BCD$  is a straight line.

**Solution**

8.



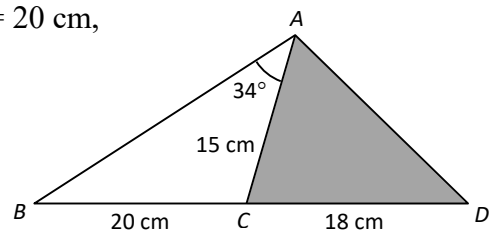
**Solution**

9. In the figure,  $BCD$  is a straight line.  $AC = 15$  cm,  $BC = 20$  cm,  $CD = 18$  cm and  $\angle BAC = 34^\circ$ . Find

- (a)  $\angle ABC$ ,  
(b) the area of  $\triangle ACD$ .

**Solution**

(a)



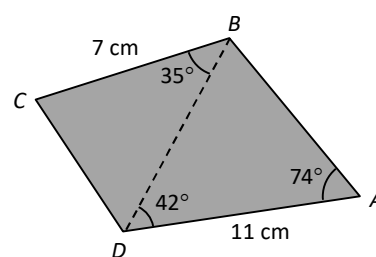
(b)

10. In the figure,  $AD = 11$  cm,  $BC = 7$  cm,  $\angle BAD = 74^\circ$ ,  
 $\angle ADB = 42^\circ$  and  $\angle CBD = 35^\circ$ .

- (a) Find the length of  $BD$ .  
(b) Find the area of quadrilateral  $ABCD$ .

**Solution**

(a)



(b)

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# Worksheet 5.3

## The Cosine Formula

NF

(Refer to Book 5A Ch5 p. 5.29 – 5.39)

Name: \_\_\_\_\_ Class: \_\_\_\_\_

### Key Points

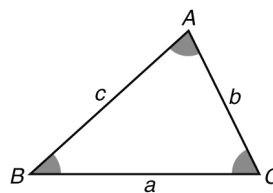
#### The cosine formula

In  $\triangle ABC$ ,

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{or} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

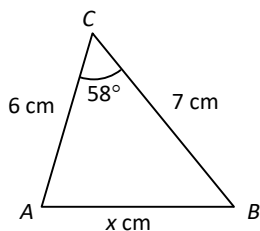
$$c^2 = a^2 + b^2 - 2ab \cos C \quad \text{or} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

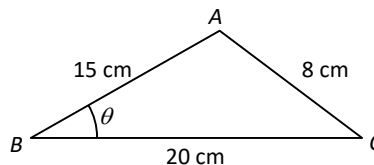
Find the unknowns in the following triangles. (1 – 2)

1.



**Solution**

2.



**Solution**



Solve  $\triangle ABC$  under the following conditions. (3 – 4)

3.  $A = 119^\circ$ ,  $b = 3.7$  cm,  $c = 3.6$  cm

**Solution**

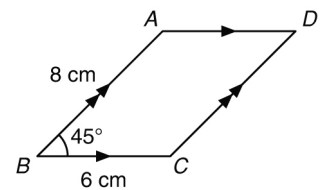
4.  $a = 11$  cm,  $b = 12$  cm,  $c = 13$  cm

**Solution**

5. In the figure,  $ABCD$  is a parallelogram with  $AB = 8$  cm,  $BC = 6$  cm and  $B = 45^\circ$ . Find the length of the diagonal  $BD$ .

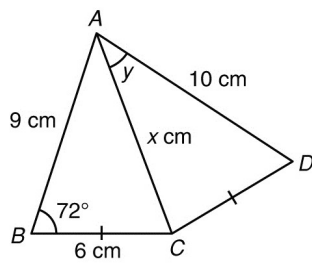
**Solution**

$$\angle BAD + (\quad) = (\quad) \quad (\text{int. } \angle\text{s, } AD \parallel BC)$$



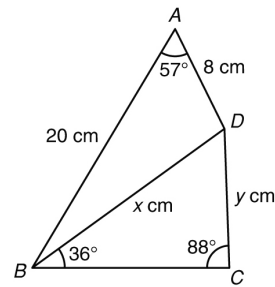
Find the unknowns in the following figures. (6 – 7)

6.



**Solution**

7.



**Solution**

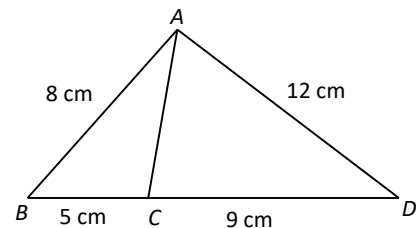
8. In the figure,  $BCD$  is a straight line.  $AB = 8$  cm,  $BC = 5$  cm,  $CD = 9$  cm and  $AD = 12$  cm.

(a) Show that  $\cos D = \frac{23}{28}$ .

- (b) Hence, find the length of  $AC$ .

**Solution**

(a)



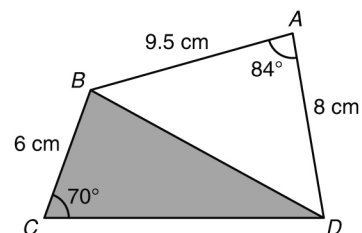
(b)

9. In the figure,  $AB = 9.5$  cm,  $BC = 6$  cm,  $AD = 8$  cm,  
 $\angle BAD = 84^\circ$  and  $\angle BCD = 70^\circ$ .

- (a) Find the length of  $BD$ .  
(b) Find the area of  $\triangle BCD$ .

**Solution**

(a)



(b)

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# Worksheet 5.4A

## Angle of Elevation and Angle of Depression

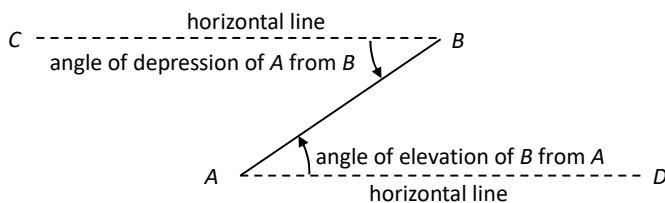
NF

(Refer to Book 5A Ch5 p. 5.39 – 5.45)

Name: \_\_\_\_\_

Class: \_\_\_\_\_

### Key Concepts



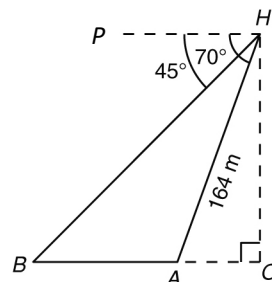
Angle of elevation of B from A = angle of depression of A from B

(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

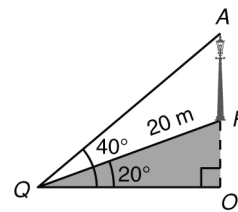
1. In the figure,  $BAO$  is a straight line on the horizontal ground.

From the top  $H$  of a lighthouse  $HO$ , the angles of depression of  $A$  and  $B$  are  $70^\circ$  and  $45^\circ$  respectively. If the distance between  $A$  and  $H$  is 164 m, find the distance between  $B$  and  $H$ .

**Solution**

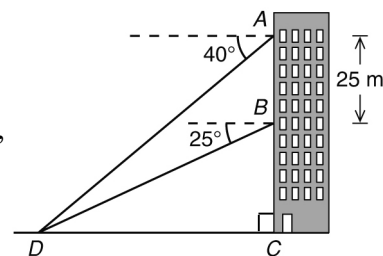


2. In the figure,  $AP$  is a lamppost standing vertically on an inclined straight road  $QP$ , which makes an angle  $20^\circ$  with the horizontal. From  $Q$ , the angle of elevation of the top  $A$  of the lamppost is  $40^\circ$ . If  $PQ = 20$  m, find the height of the lamppost.



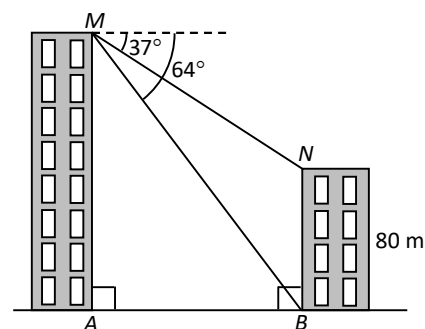
**Solution**

3. In the figure, a police car  $D$  is observed by two policemen  $A$  and  $B$  in the police station. The angles of depression of  $D$  from  $A$  and  $B$  are  $40^\circ$  and  $25^\circ$  respectively. If the distance between  $A$  and  $B$  is 25 m, find the distance between the police car and the police station.



**Solution**

4. In the figure,  $MA$  and  $NB$  are two buildings on the horizontal ground. The angles of depression of  $N$  and  $B$  from  $M$  are  $37^\circ$  and  $64^\circ$  respectively. If the height of building  $NB$  is 80 m, find
- (a) the distance between  $M$  and  $B$ ,
  - (b) the height of building  $MA$ .

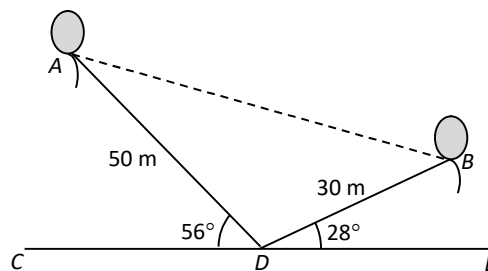


**Solution**

(a)

(b)

5. In the figure, two balloons, fastened by two taut strings  $AD$  and  $BD$ , are vertically above a straight road  $CDE$ . The lengths of the strings  $AD$  and  $BD$  are 50 m and 30 m respectively. It is given that  $\angle ADC = 56^\circ$  and  $\angle BDE = 28^\circ$ .



- (a) Find the distance between  $A$  and  $B$ .  
(b) Find the angle of depression of  $B$  from  $A$ .

**Solution**

(a)

(b)

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# Worksheet 5.4B

## True Bearing and Compass Bearing

NF

(Refer to Book 5A Ch5 p. 5.45 – 5.51)

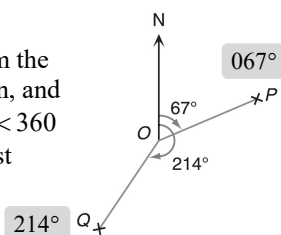
Name: \_\_\_\_\_

Class: \_\_\_\_\_

### Key Concepts

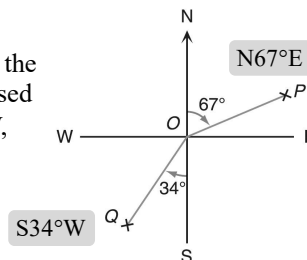
#### True bearing

Directions are measured from the north in a clockwise direction, and expressed as  $x^\circ$ , where  $0 \leq x < 360$  and the integral part of  $x$  must consist of 3 digits.



#### Compass bearing

Directions are measured from the north or the south, and expressed as  $Nx^\circ E$ ,  $Nx^\circ W$ ,  $Sx^\circ E$  or  $Sx^\circ W$ , where  $0 < x < 90$ .



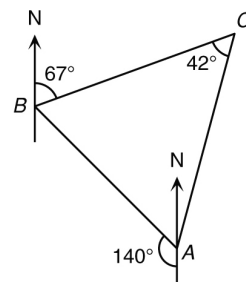
(In this worksheet, numerical answers should be either exact or correct to 3 significant figures.)

1. In the figure,  $A$ ,  $B$  and  $C$  lie on the same horizontal plane.

(a) Find the compass bearing of  $A$  from  $B$ . \_\_\_\_\_

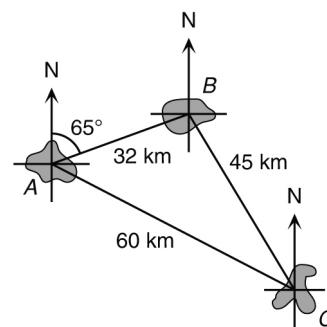
(b) Find the compass bearing of  $B$  from  $C$ . \_\_\_\_\_

(c) Find the true bearing of  $C$  from  $A$ . \_\_\_\_\_



2. The figure shows three islands  $A$ ,  $B$  and  $C$ . The true bearing of  $B$  from  $A$  is  $065^\circ$ . It is known that  $AB = 32$  km,  $BC = 45$  km and  $CA = 60$  km. Find the true bearing of  $C$  from  $B$ .

**Solution**





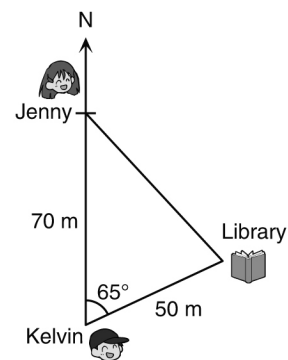
3. As shown in the figure, Kelvin is 50 m from a library. The compass bearing of the library from Kelvin is  $N65^\circ E$ . Jenny is 70 m due north of Kelvin. Find

- (a) the distance between Jenny and the library,  
(b) the compass bearing of the library from Jenny.

**Solution**

(a)

(b)



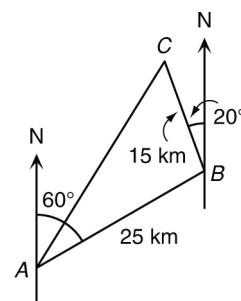
4. In the figure, a ship leaves port  $A$  and sails 25 km in the direction of  $N60^\circ E$  to port  $B$ . It then sails 15 km in the direction of  $N20^\circ W$  to port  $C$ .

- (a) Find the distance between port  $A$  and port  $C$ .  
(b) Find the compass bearing of port  $A$  from port  $C$ .

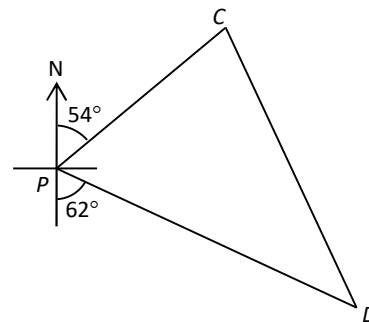
**Solution**

(a)

(b)



5. At 10 a.m., ships  $A$  and  $B$  depart from a port  $P$ . Ship  $A$  sails at 24 km/h on a course of  $N54^\circ E$  and ship  $B$  sails at 30 km/h on a course of  $S62^\circ E$ . At noon, ships  $A$  and  $B$  arrive at  $C$  and  $D$  respectively.



- (a) Find the distance between  $C$  and  $D$ .
- (b) Find  $\angle PCD$ .
- (c) If ship  $A$  then sails from  $C$  to  $D$ , what is the shortest distance between ship  $A$  and the port  $P$  during this journey?

**Solution**

(a)

(b)

(c)